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MATHEMATICAL MODELING AND OPTIMIZATION OF TRAFFIC FLOWS

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Abstract.

The article presents a mathematical model for optimizing traffic flow in an urban environment based on a stochastic approach. It allows to optimize traffic flows using a genetic algorithm by changing the phases of traffic lights operation. An exponential law of distribution of the generation of cars at the input points of the transport network has been established. The relationship between the intensity of servicing the traffic flow and the time of the green signal of the traffic light is revealed. Practical calculation have confirmed the applicability of the optimization model in traffic management.

Introduction

Traffic flows management in the urban environment is one of the urgent tasks for the smart city system functioning. The solution to this problem affects the quality of life of the population and affects the interests of the vast majority of citizens.

The process of traffic flows management is a topical object of research, the attention to which is growing in proportion to the level of motorization in the world. The severity of the transport problem requires a systematic approach to its solution.

Two main approaches in the formulation of the problem of traffic management optimization are used: **deterministic and stochastic** ,which are aimed at solving the problem of adaptive regulation of the cycle of traffic lights, researching stable modes of their operation, joint consideration of several intersections. Thus, to solve the existing problem of traffic managements, it is necessary to expand the base of applied method, including through the development of mathematical models of traffic flows.

There are interdisciplinary mathematical methods and models in the researches of traffic flows in an urban environment, including methods and models are based on the probabilistic approach. Their feasibility is justified by the presence of network restrictions and the massive nature of the traffic flow. Therefore, it is possible to identify clear patterns in the formation of queues at intersections, intervals, loads along the road lanes,etc…

Because the mathematical model of traffic flows has a complex stochastic structure, it is necessary to use heuristic algorithms to optimize urban traffic flows.

The purpose of this research is to develop mathematical models for solving the problem of traffic flows optimizing in an urban environment using a genetic algorithm.

Mathematical Modeling of traffic flows

Input Flows Modeling

The input flow is the flow of cars approaching the intersection in one of the directions. For creating of a mathematical model of the input flow, which will be closed to reality, it is necessary to establish the distribution of the interval of cars to the intersection.

To establish the distribution law of the input cars flow, experiments were carried out with fixing the time between approaching cars for two intersections in the city of Tamil nadu :

- the intersection of Periyar street and Annai street (flow from the eastern side of the crossroads);
- the interaction of Anna street and Annai street (flow from the southern side of the crossroads).

At each interaction, the time of arrival of each car at a distance of 30 meters to the intersection was recorded. As a result of measurements, two sample sets were obtained with a time fixation accuracy of 0.001 sec.

Figure 1 and figure 2 show the histogram of the distribution of relative frequencies for the interval series of both selections.

The presented histograms of relative frequencies are comparable to the graph of the density of the exponential distribution, therefore, the corresponding null hypotheses were put forward H_0.

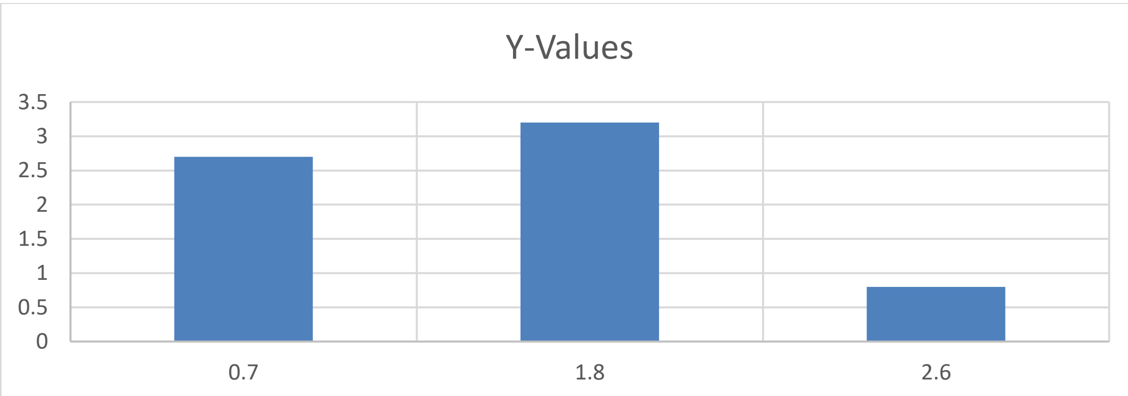


Figure 1. Histogram for selection 1(Periyar street and Annai street).

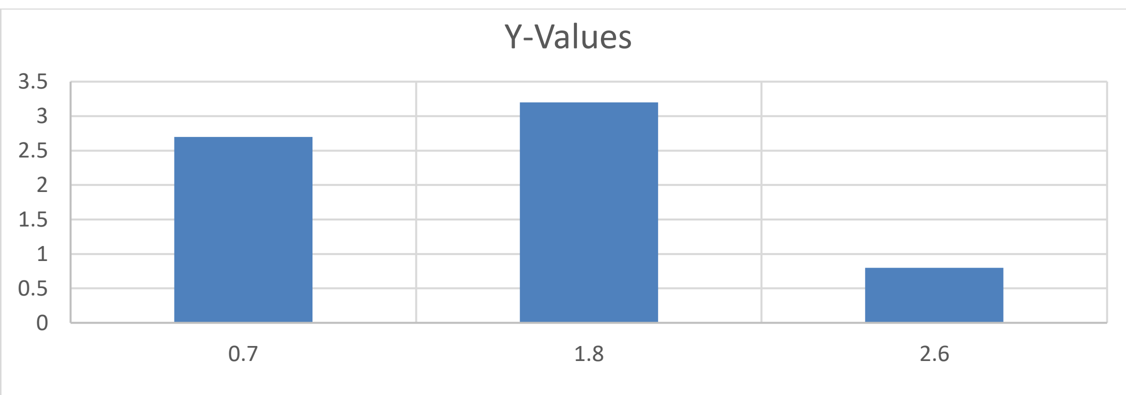


Figure 2. Histogram for selection 2 (Anna street and Annai street)

Table 1 indicates the results of estimating the parameters of the distribution of input flows and testing statistical hypotheses about the exponential distribution of selections. The coefficients of variation (see Table 1) of the samples are comparable with the corresponding parameter of the exponential distribution (V-1) and do not contradict the hypotheses put forward.

Table 1. parameters of input transport flows.

Parameter name	Designation	Periyar St. and Annai St.	Anna St. and Annai St.
Mean,sec.	**X**	3.728	4.864
Dispersion,sec.^{2}	D	12.957	21.500
standard deviation,sec.	σ	3.600	4.637

variation coefficient	**V=σ/X**	0.966	0.953
Intensity, 1/sec.	**λ**=σ/X	0.268	0.206
Significan-ce level	**α**	0.05	0.05
Critical point	χ_{α}^{2}	12.6	12.6
Observed value of pearson's criterion	**χ =Σ**	8.84	8.84
Acceptance condition	**χϵ(0; χ_{α}^{2})**	8.84 (0;12.6)	4.85 (0;12.6)

Hypothesis test result is sampling selections have exponential distribution with parameters λ_1 -0..268 and λ_2 -0.206. Similar calculations were carried out for various intersections at different times of the day. The exponential distribution hypothesis is confirmed in 80% of cases.

Consequently, the car flow has a distribution close to the exponential law, therefore, in the simulation model, the time interval between the cars are generated randomly according to the exponential distribution.

Dependence of the Traffic Flow Service Intensity on the Traffic Light Operating Mode

The number of cars, that can pass through the interaction per unit of time, is determined by the length of the path traveled by the first car in the queue at the traffic light (see below diagram).

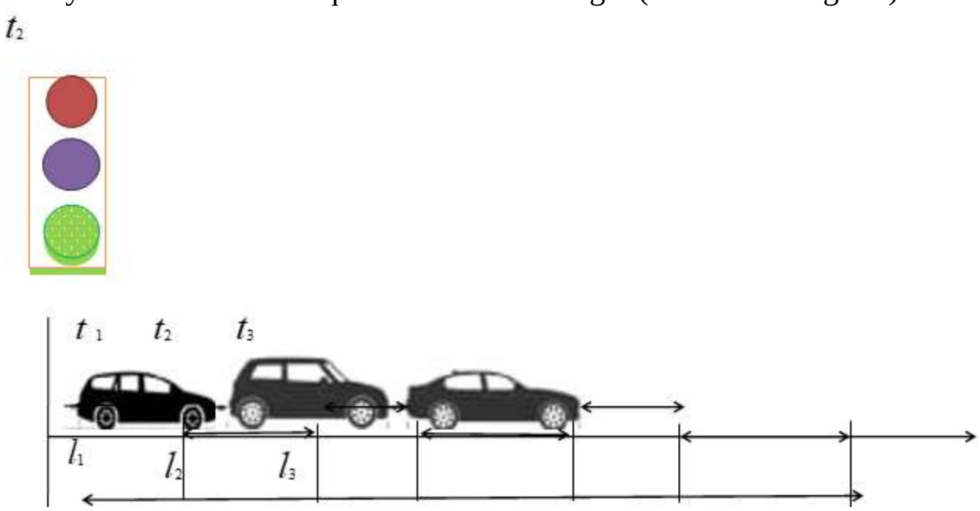


Figure 3. Diagram of cars that crossed the stop line of the traffic light.

The intensity of car traffic during the operation of the green traffic light $\mu_1(\tau)$ is not a constant function and there are 3 period :

- From the moment beginning of the first car in the queue until the flow of the recommended speed is reached;
- From the moment the recommended speed is reached until the last car in the queue has passed;
- From the moment of passing the queue to the end of the green traffic light .

Traveled first car distance S in the queue during the time of the green signal of the traffic light τ when the cars are accelerating is determined by the formula:

$$S = a\tau^2/2 \quad (1)$$

The number of cars passed during the time τ is:

$$N = \frac{S}{l_0 + l_a}$$

$$N = \frac{a\tau^2}{2(l_0+l_a)} \quad (2)$$

Whence the intensity of car maintenance for the first period of operation of the green traffic light is calculated by the fornula:

$$\mu_1(\tau) = \frac{a\tau}{2(l_0+l_a)} \quad (3)$$

Time of transition from the first to the second period of the green traffic signal is:

$$\tau^* = \frac{V}{a} \quad (4)$$

Similarly, we find the intensity for the second period of the green traffic signal:

$$S = S^*+V(\tau - \tau^*) . \qquad (5)$$

$$S^* = a\tau^{*2}/2$$

$$S^* = V^2/2a \qquad (6)$$

$$N = \frac{V^2}{2a(l_0+l_a)} + \frac{V(\tau-\tau^*)}{l_0+l_a} \quad (7) \qquad \mu_2(\tau) = \frac{V^2}{2a\tau(l_0+l_a)} + \frac{V(\tau-\tau^*)}{\tau(l_0+l_a)}$$

(8)

Time of transition from the second to the third period of the green trafic signal:

$$\mu(\tau^{**}) = \lambda \qquad (9)$$

$$\left(\frac{V^2}{2a(l_0+l_a)} + \frac{V(\tau^{**}-\tau^*)}{l_0+l_a}\right) / \tau^{**} = \lambda . \qquad (10)$$

$$\tau^{**} = \frac{V^2}{2a(V-\lambda(l_0+l_a))} \quad (11)$$

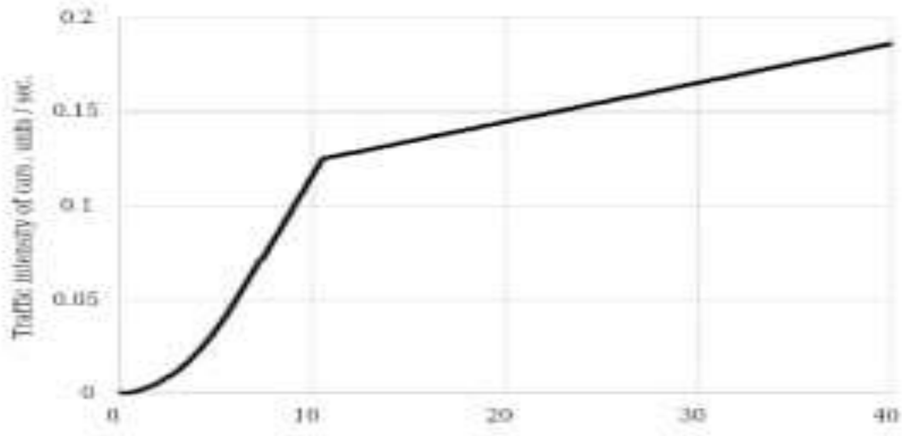
Further, we find the intensity for the third of the green traffic signal:

$$S = S^{**}+\lambda(\tau-\tau^{**})(l_0+l_a) \quad (12) \qquad \mu_3(\tau) = \frac{V^2}{2a\tau(l_0+l_a)} + \frac{V(\tau^{**}-\tau^*)}{\tau(l_0+l_a)} + \frac{\lambda(\tau-\tau^{**})}{\tau} \quad (13)$$

A piecewise continuous function of the traffic flow service intensity is obtained depending on the operating time of the green traffic light:

$$\mu_0(\tau) = \begin{cases} a\tau / 2(l_0+l_a), \tau \in [0;\tau^*] \\ (V^2/2a + V(\tau-\tau^*))/\tau(l_0+l_a), \tau \in (\tau^*;\tau^{**}] \\ (V^2/2a + V(\tau^{**}-\tau^*))/(l_0+l_a) + \lambda(\tau-\tau^{**})/\tau, \tau \in (\tau^{**};T] \end{cases} \quad (14)$$

The figure 4 given in below shows a graph of the traffic intensity of cars during the operation of the green traffic light.



The duration of the green signal of the traffic light, sec. **Figure 4.** The intensity of car traffic during the time of the green traffic light.

This graph shows that the traffic intensity of cars during the first phase changes parabolic, during the second phase -linearly to the end of the queue, and the intensity in the third phase depends on the input flow.

Conclusion

A mathematical model of traffic flows based on a stochastic approach has been developed, which makes it possible to assess the change in the congestion of sections of the transport network over time.

Using the developed mathematical model, traffic flows are optimized by changing the phase of traffic lights based on the use of a genetic algorithm.

An exponential distribution law of the appearance of cars is established on the basis of two input points of car generation. A similar study was carried out for many points of vehicle generation and the exponential distribution law was confirmed in 80%.

The analytical relationship between the intensity of servicing the traffic flow and the time of the green signal of the traffic lights is revealed. Practical calculations have confirmed the appllicability of the optimization model in traffic management.

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